All You Wanted to Know About Quantum Programming Without Daring to Ask

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Plan

Quantum Computation

Design of Quantum Programming Languages

Quantum Computation

Model of Computation Overview of Quantum Algorithms Case Study

Design of Quantum Programming Languages

Plan

Quantum Computation Model of Computation

Overview of Quantum Algorithms Case Study

Design of Quantum Programming Languages



Riesebos, L., et al. "Quantum Accelerated Computer Architectures." Proc. IEEE International Symposium on Circuits and Systems (ISCAS), 2019.

Leading technologies in NISO era1 Candidate technologies beyond NISO Qubit type or Superconducting² Trapped ion Photonic Silicon-based³ Topological⁸ technology Description of Nuclear or electron spin or charge of aubit encoding of single photons 6×39 2 Physical gubits45 ŕ Oubit lifetime ~50–100 us ~50 s ~150 µs ~1-10 s ᠿ Gate fidelity7 ~99.4% ~99.9% ~98% ~90% target ~99.9999% Gate operation ~10-50 ns ~3-50 µs ~1 ns ~1-10 ns ര time × Connectivity Nearest demonstrated ?) Scalability Single photon Maturity or 10 technology readiness level TRL 3 TRL 3 TRL 1 Improves with Room Cryogenic Estimated: Cryogenic operation cryogenic temperature operation **a**--- 1 Key properties Long lifetime Fast gating temperatures Fast gating Fast gating Silicon technology Long qubit lifetime High fidelities Modular design Atomic-scale size Vacuum operation

From 2018 : https://www.bcg.com/publications/2018/next-decade-quantum-computing-how-play

NISQ era

- Noisy Intermediate Scale Quantum
- Small-to-medium memory sizes, noisy
- ► Tradeoffs: Number of Qubit, Noise/Fidelity, Connectivity.
- Challenge: Emulation! (with Tensor Network, etc)

LSQ era

- Large-Scale Quantum
- Stabilized, logical qubits
- Tradeoffs: Error Correction, Layout, Compilation
- Challenges: Hardware!



Roadmap from 2022

What **COULD** quantum algorithms be good for?

- factoring
 - for breaking modern cryptography
- simulating quantum systems
 - for more efficient molecule distillation procedure
- solving linear systems
 - for high-performance computing
- solving optimization problems
 - for big learning
- ▶ ... more than 300 algorithms:

http://math.nist.gov/quantum/zoo/

Dichotomy between

- Quantum algorithms as theoretical tools for complexity analysis
- Quantum algorithms as practical tools for concrete problems

Challenges, assuming that a physical machine is available

- Designing the right computational model
- Moving from mathematical representation to code
- Resource estimation, optimization
- Compilation and low-level representation
- Debugging/unit testing hard : code analysis and verification







The program lives here



This only holds the quantum memory



Series of instructions/feedbacks

- Sequential stream of local instructions
- Updating the memory: Reversible, unitary operations
- Reading the memory: probabilistic, destructive measures



No "quantum loop" or "conditional escape".

A quantum register with *n* quantum bits is a complex combination of strings of *n* bits in a Hilbert space. E.g. for n = 3:

$$\begin{array}{rrr} & -\frac{1}{2} \cdot |0 \ 0 \ 0 \rangle \\ + & \frac{1}{2} \cdot |0 \ 0 \ 1 \rangle \\ + & \frac{1}{2} \cdot |1 \ 1 \ 0 \rangle \\ - & \frac{1}{2} \cdot |1 \ 1 \ 1 \rangle \end{array}$$

with a norm condition.

The state of an *n*-qubit register lives in $\mathcal{H}_n \triangleq \mathbb{C}^{2^n}$.

- \rightarrow vectors of dimension 2^n .
- \rightarrow basis elements: bitstrings of size *n*.

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & |0\,0\rangle \\ + & \frac{i}{\sqrt{2}} & |1\,1\rangle \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} & |0\,0\,1\rangle \\ + & \frac{i}{\sqrt{2}} & |1\,0\rangle \\ + & \frac{i}{\sqrt{2}} & |0\,1\,0\rangle \end{pmatrix}$$

$$\begin{array}{c|c} \frac{1}{\sqrt{2}} & |0\,0\rangle \otimes \left(\begin{array}{ccc} \frac{1}{\sqrt{2}} & |0\,0\,1\rangle \\ + & \frac{i}{\sqrt{2}} & |1\,0\,0\rangle \\ + & \frac{i}{\sqrt{2}} & |0\,1\,0\rangle \end{array}\right) \\ + & \frac{i}{\sqrt{2}} & |0\,0\,1\rangle \\ + & \frac{i}{\sqrt{2}} & |0\,0\,1\rangle \\ + & \frac{i}{\sqrt{2}} & |0\,1\,0\rangle \end{array}$$



$$\begin{array}{c|c} \frac{1}{2} & |0\,0\,0\,0\,1\rangle \\ \frac{1}{2} & |0\,0\,1\,0\,0\rangle \\ \frac{1}{2} & |0\,0\,0\,1\,0\rangle \\ + & \frac{1}{2} & |1\,1\,0\,01\rangle \\ \frac{-1}{2} & |1\,1\,1\,0\,0\rangle \\ \frac{-1}{2} & |1\,1\,0\,10\rangle \end{array}$$

The joint state of two quantum registers of sizes m and n lives in the tensor product space $\mathcal{H}_m \otimes \mathcal{H}_n$.

$$\begin{array}{c|c} \frac{1}{2} & |0\ 0\ 0\ 0\ 1\rangle \\ \frac{1}{2} & |0\ 0\ 1\ 0\rangle \\ \frac{1}{2} & |0\ 0\ 0\ 1\ 0\rangle \\ + & \frac{1}{2} & |1\ 1\ 0\ 0\rangle \\ \frac{-1}{2} & |1\ 1\ 0\ 0\rangle \\ \frac{-1}{2} & |1\ 1\ 0\ 1\rangle \end{array}$$

 $\mathcal{H}_m \otimes \mathcal{H}_n$ is of dimension $2^m \times 2^n$.

Takeaway

- Classical data in superposition.
- Internal updates are reversible and local.
- No-cloning theorem: quantum information cannot be copied
- Probablistic, destructive reading
- No control flow in the quantum co-processor
- Some parallelism thanks to data superposition

Plan

Quantum Computation Model of Computation Overview of Quantum Algorithms Case Study

Design of Quantum Programming Languages

Structure of Quantum Algorithms



Structure of Quantum Algorithms

Simple scheme



General scheme



- Quantum circuits \neq hardware design
- Hybrid classical/quantum computation

- 1. Quantum primitives.
 - Quantum Fourier Transform.
 Assuming ω = 0.xy, we want

$$(e^{2\pi i\omega})^{0} \cdot 00$$

$$+ (e^{2\pi i\omega})^{1} \cdot 01$$

$$+ (e^{2\pi i\omega})^{2} \cdot 10$$

$$+ (e^{2\pi i\omega})^{3} \cdot 11$$

$$\longrightarrow 1 \cdot xy$$

- 1. Quantum primitives.
 - Quantum Fourier Transform.
 - Amplitude amplification.
 Qubit 3 in state 1 means good.

$$\begin{array}{cccc} & \alpha_0 \cdot 000 & & \alpha_0 \cdot 000 \\ + & \alpha_1 \cdot 011 & + & \alpha_1 \cdot 011 \\ + & \alpha_2 \cdot 100 & + & \alpha_2 \cdot 100 \\ + & \alpha_3 \cdot 110 & + & \alpha_3 \cdot 110 \end{array}$$

- 1. Quantum primitives.
 - Quantum Fourier Transform.
 - Amplitude amplification.
 - Quantum walk.



The techniques used to described quantum algorithms are diverse.

- 1. Quantum primitives.
 - Quantum Fourier Transform.
 - Amplitude amplification.
 - Quantum walk.

After 5 steps of a probabilistic walk:



The techniques used to described quantum algorithms are diverse.

- 1. Quantum primitives.
 - Quantum Fourier Transform.
 - Amplitude amplification.
 - Quantum walk.

After 5 steps of a quantum walk:



The techniques used to described quantum algorithms are diverse.

2. Oracles.

▶ Take a classical function $f : Bool^n \to Bool^m$.

Construct

$$egin{array}{rcl} \overline{f} : & ext{Bool}^{n+m} & \longrightarrow & ext{Bool}^{n+m} \ & (x,y) & \longmapsto & (x,y\oplus f(x)) \end{array}$$

• Build the unitary U_f acting on n + m qubits computing \overline{f} .

The techniques used to described quantum algorithms are diverse.

3. Blocks of loosely-defined low-level circuits.



This is not a formal specification!

- 4. High-level operations on circuit:
 - Circuit inversion.



- 5. Classical processing.
 - Generating the circuit...
 - Computing the input to the circuit.
 - Processing classical feedback in the middle of the computation.
 - Analyzing the final answer (and possibly starting over).

Quantum Computation

Model of Computation Overview of Quantum Algorithms Case Study

Design of Quantum Programming Languages
Considering a vector \vec{b} and the system

$$A \cdot \vec{x} = \vec{b},$$

compute the value of $\langle \vec{x} | \vec{r} \rangle$ for some vector \vec{r} .

Practical situation: the matrix *A* corresponds to the finite-element approximation of the scattering problem:

arXiv:1505.06552

Three oracles:

- ▶ for \vec{r} and for \vec{b} : input an index, output (the representation of) a complex number
- ▶ for A: input two indexes, output also a complex number

Many quantum primitives

- Amplitude estimation
- Phase estimation
- Amplitude amplification
- Hamiltonian simulation



- Yellow: Elementary gates.
- Red: Oracles.
- Blue: QFT's.
- Black: Subroutines.
- Parameters:

Dimensions of the space; Precision for each of the vectors; Allowed error; Various parameters for *A*... In total, 19 parameters.

Oracle R is given by the function

```
calcRweights y nx ny lx ly k theta phi =
    let (xc',yc') = edgetoxy y nx ny in
    let xc = (xc'-1.0)*lx - ((fromIntegral nx)-1.0)*lx/2.0 in
   let yc = (yc'-1.0)*ly - ((fromIntegral ny)-1.0)*ly/2.0 in
   let (xg,yg) = itoxy y nx ny in
    if (xg == nx) then
       let i = (mkPolar ly (k*xc*(cos phi)))*(mkPolar 1.0 (k*yc*(sin phi)))*
                ((sinc (k*ly*(sin phi)/2.0)) :+ 0.0) in
       let r = (\cos(phi) :+ k*lx)*((\cos(theta - phi))/lx :+ 0.0) in i * r
   else if (xg==2*nx-1) then
        let i = (mkPolar ly (k*xc*cos(phi)))*(mkPolar 1.0 (k*yc*sin(phi)))*
                ((sinc (k*ly*sin(phi)/2.0)) :+ 0.0) in
       let r = ( cos(phi) :+ (- k*lx))*((cos (theta - phi))/lx :+ 0.0) in i * r
   else if ( (yg==1) && (xg<nx) ) then
       let i = (mkPolar lx (k*yc*sin(phi)))*(mkPolar 1.0 (k*xc*cos(phi)))*
                ((sinc (k*lx*(cos phi)/2.0)) :+ 0.0) in
       let r = ( (- sin phi) :+ k*ly )*((cos(theta - phi))/ly :+ 0.0) in i * r
   else if ( (yg==ny) & (xg<nx) ) then
        let i = (mkPolar lx (k*yc*sin(phi)))*(mkPolar 1.0 (k*xc*cos(phi)))*
                ((sinc (k*lx*(cos phi)/2.0)) :+ 0.0) in
       let r = ( (- sin phi) :+ (- k*ly) )*((cos(theta - phi)/ly) :+ 0.0) in i * r
   else 0.0 :+ 0.0
```

The algorithms create circuits whose sizes and shapes depend on the parameters. E.g. the size of the input register:



(QFT)

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(diffusion step in BWT)

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(piece of one subroutine of QLS)

The algorithms create circuits whose sizes and shapes depend on the parameters. E.g. the size of the input register:



(the subroutine U_g)

The algorithms create circuits whose sizes and shapes depend on the parameters. E.g. the size of the input register:



(the subroutine U_b)

Plan

Quantum Computation

Design of Quantum Programming Languages

Accessing Qubits Handling Parametricity

Conclusion

Lessons learned

- Circuit construction
 - Procedural: Instruction-based, one line at a time
 - Declarative: Circuit combinators
 - Inversion
 - Repetition
 - Control
 - Computation/uncomputation
- Circuits as inputs to other circuits
- Regularity with respect to the size of the input
- Distinction parameter / input
- Need for automation for oracle generation

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Programming framework

Two approaches

- Circuit as a record
 - One type circuit
 - Qubits = wire numbers
 - Native: vertical/horizontal concatenation, gate addition

Circuit as a function

- Qubits \equiv first-order objects
- Input wires ≡ function input
- Output wires \equiv function output

Circuits as Records

Simplest model: an object holding all of the circuit structure

- Classical wires
- Quantum wires
- List of gates (or directed acyclic graph)
- This is for instance QisKit/QASM model

In this system

- Static circuit
- ► No high-level hybrid interaction: sequence
 - 1. circuit generation
 - 2. circuit evaluation
 - 3. measure
 - 4. classical post-processing
 - 5. back to (1)

Circuits as Records

Procedural construction (QisKit)

```
q = QuantumRegister(5)
c = ClassicalRegister(1)
circ = QuantumCircuit(q,c)
```

```
circ.h(q[0])
for i in range(1,5):
    circ.cx(q[0], q[i])
circ.meas(q[4],c[0])
```

- Static ID For registers
- Wires are numbers
- Gate \equiv instruction
- Classical control: Circuit building
- Explicit "run" of circuit

Combinators: return a record circuit

- circ.control(4)
- circ.inverse()
- circ.append(other-circuit)

A function

a -> Circ b

- Inputs something of type a
- Outputs something of type b
- As a side-effect, generates a circuit snippet.

Or

- Inputs a value of type a
- Outputs a computation of type b

The circuit



can be typed with

Qubit -> Circ (Qubit,Qubit)

- Inputs one qubit
- Outputs a pair of qubits
- Spits out some gates when evaluated

The gates are however encapsulated in the function

Representing circuits (Quipper)



```
Procedural presentation of circuits:
```

```
prog :: Qubit -> Circ (Qubit,Qubit)
prog q = do
    hadamard_at q
    r <- qinit False
    qnot_at r 'controlled' q
    return (q,r)</pre>
```

```
Procedural presentation of circuits:
```

```
Procedural presentation of circuits:
```



```
Procedural presentation of circuits:
```



```
Procedural presentation of circuits:
```





```
import Quipper
circ ::
    Qubit -> Circ (Qubit,Qubit)
circ x = do
    y <- qinit False
    hadamard_at x
    qnot_at y 'controlled' x
    return (x,y)
```

• Qubits \equiv first-class variable

- ► Circuit ≡ function
- Wires \equiv inputs and outputs
- Mix classical/quantum

Wires do not have "fixed" location

```
circ2 :: Qubit -> Circ ()
circ2 x = do
    (x1,x2) <- circ x
    (y1,y2) <- circ x1
    (z1,z2) <- circ x2
    return ()</pre>
```



- Qubit \neq Wire number
- Circuits as functions: can be applied
- More expressive types

Controls

controlled :: ControlSource b => Circ a -> b -> Circ a

Input: Computation generating a circuit C

- Input: Something that can be controlled (e.g. Qubit)
- Output: Computation generating the controlled circuit C

Example

Controls

controlled :: ControlSource b => Circ a -> b -> Circ a

Input: Computation generating a circuit C

- Input: Something that can be controlled (e.g. Qubit)
- Output: Computation generating the controlled circuit C

Example

```
prog :: (Qubit,Qubit) -> Circ (Qubit,Qubit)
prog (p,q) = do
    controlled (qnot_at p) q
    return (p,q)
```

Controls

controlled :: ControlSource b => Circ a -> b -> Circ a

Input: Computation generating a circuit C

- Input: Something that can be controlled (e.g. Qubit)
- Output: Computation generating the controlled circuit C

Example

```
prog :: (Qubit,Qubit) -> Circ (Qubit,Qubit)
prog (p,q) = do
    qnot_at p 'controlled' q
    return (p,q)
```

with infix notation

Controls

```
controlled :: ControlSource b => Circ a -> b -> Circ a
```

```
It works on any (reversible) circuit
prog :: (Qubit,Qubit) -> Circ (Qubit,Qubit)
prog (p,q) = do
    hadamard_at p
    hadamard_at q
    qnot_at p 'controlled' q
    return (p,q)
```

Controls

```
controlled :: ControlSource b => Circ a -> b -> Circ a
```

Controls

```
controlled :: ControlSource b => Circ a -> b -> Circ a
```

```
It works on any (reversible) circuit
prog :: (Qubit,Qubit) -> Circ (Qubit,Qubit)
...
prog2 :: (Qubit,Qubit,Qubit) -> Circ ()
prog2 (p,q,r) = do
    prog (p,q)
    prog (q,r) 'controlled' p
    prog (p,r)
    return ()
```

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Conclusion

A program

- Inputs classical parameters
- Construct a circuit from these parameters
- Run the circuit

Circuits: parametrized families

Example: QFT



Example: QFT



Example: QFT


Families of Circuits

With the help of lists:



Families of Circuits

```
List combinators, e.g.
```

```
mapM :: (a -> Circ b) -> [a] -> Circ [b]
```

```
Mixed presentation of circuits:
```

List of size 2:

```
prog :: Qubit -> Circ (Qubit,Qubit)
prog q = do
    hadamard_at q
    r <- qinit False
    qnot_at r 'controlled' q
    return (q,r)
prog2 :: [Qubit] -> Circ [(Qubit,Qubit)]
```

prog2 l = mapM prog l



Families of Circuits

```
List combinators, e.g.
```

```
mapM :: (a -> Circ b) -> [a] -> Circ [b]
```

```
Mixed presentation of circuits:
```

```
prog :: Qubit -> Circ (Qubit,Qubit)
prog q = do
    hadamard_at q
    r <- qinit False
    qnot_at r 'controlled' q
    return (q,r)</pre>
```

```
prog2 :: [Qubit] -> Circ [(Qubit,Qubit)]
prog2 l = mapM prog l
```



Example: BWT

```
import Quipper
```

```
w :: (Qubit,Qubit) -> Circ (Qubit,Qubit)
w = named_gate "W"
```

```
toffoli :: Qubit -> (Qubit,Qubit) -> Circ Qubit
toffoli d (x,y) =
  qnot d 'controlled' x .==. 1 .&&. y .==. 0
                                                        W
                                                                                     W^{\dagger}
eiz_at :: Qubit -> Qubit -> Circ ()
eiz_at d r =
  named_gate_at "eiZ" d 'controlled' r .==. 0
                                                        W
                                                                                     W^{\dagger}
circ :: [(Qubit,Qubit)] -> Qubit -> Circ ()
circ ws r = do
                                                   a2n -
                                                        W
                                                                                     W^{\dagger}
  label (unzip ws,r) (("a","b"),"r")
  d <- qinit 0
  mapM_ w ws
  mapM_ (toffoli d) ws
  eiz_at d r
  mapM_ (toffoli d) (reverse ws)
  mapM_ (reverse_generic w) (reverse ws)
  return ()
```

main = print_generic EPS circ (replicate 3 (qubit,qubit)) qubit

Example: BWT

Result (3 wires):



Example: BWT

Result (30 wires):



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Conclusion

Quantum Computation and Programming

- A lot of classical programming!
- Many challenges, both at high and low-level
- Research active to match theory with practice.

The QuaCS team at LMF:

- Design of quantum programming languages
- Model of quantum computation
- Compilation toolchain
- Circuit synthesis and optimization
- Intermediate representation
- Code certification

